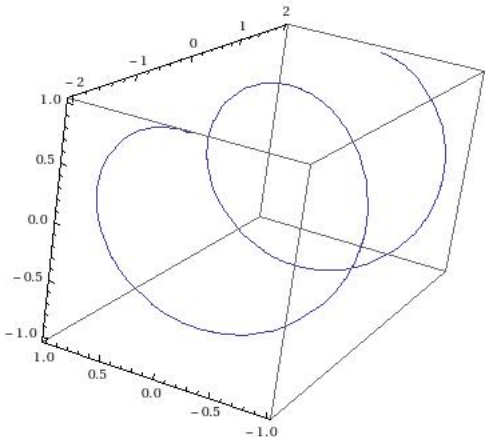
Lessons 1-4 Review Problems

June 21, 2016

1. Write inequalities to describe the region consisting of all points on or between spheres of radius r and R centered at the origin, where $r \leq R$. What if the sphere is centered at $(1, 0, -5)$?
2. Find a unit vector in the direction of $\vec{a} = \langle 1, 1, 1 \rangle$.
3. (a) If $\vec{a} \cdot \vec{b} = 8$, $|\vec{a}| = 4$ and $|\vec{b}| = 4$, what is the angle in radians between $\vec{a}$ and $\vec{b}$?
   (b) If $\vec{a} \times \vec{b} = \langle 2, 0, 2 \rangle$, $|\vec{a}| = 4$ and $|\vec{b}| = 4$, what is the angle in radians between $\vec{a}$ and $\vec{b}$?
4. Write a vector equation of the line segment from $(1, -5, 0)$ to $(5, 4, -10)$.
5. At what point(s) does the line $\vec{r}(t) = \langle t, 2t, 3t \rangle$ intersect the surface $2x + y - z = 10$?
6. Find the equation of the plane perpendicular to both $3x + y - z = 0$ and $x - z = 10$ which goes through the point $(1, 0, 2)$.
7. (a) Identify and sketch the quadric surface $x^2 + 4y^2 + 2y - z^2 = 0$.
   (b) Identify and sketch the quadric surface $x^2 + 4y^2 + 2y - z^2 = -1/4$.
8. Sketch the curve parametrized by $\vec{r}(t) = \langle \frac{t}{\pi}, \sin t, \cos t \rangle$. Indicate the direction of the curve with arrows and label $\vec{r}(0)$, $\vec{r}(\pi)$, and $\vec{r}(-\pi)$.
9. If $\vec{r}(t) = \langle 2te^{3t}, 0, t^2 \rangle$, find $\vec{r}'(t)$ and $\int_0^1 \vec{r}(t)dt$.
10. Find the arc length function, $s(t)$, for $\vec{r}(t) = \langle 2t, 2t^2, \frac{8}{3}t^{3/2} \rangle$ for $t \geq 0$.
11. Find $\vec{T}(t)$, $\vec{N}(t)$, and the curvature of $\vec{r}(t) = \langle 2t^2, t, 3t \rangle$.

Answers

1. $\{(x,y,z)|r^2 \leq x^2+y^2+z^2 \leq R^2\}$; $\{(x,y,z)|r^2 \leq (x-1)^2+y^2+(z+5)^2 \leq R^2\}$
2. $\frac{1}{\sqrt{3}}\langle 1,1,1\rangle$
3. (a) $\frac{\pi}{3}$
   (b) $\frac{\pi}{6}$
4. $\vec{r}(t) = \langle 1+4t, -5+9t, -10t\rangle$, $0 \leq t \leq 1$ (there are other solutions, too!)
5. $(10,20,30)$
6. $(x-1)-2y+(z-2)=0$ (or $x-2y+z=3$)
7. (a) hyperboloid of one sheet centered at $(0,-1/4,0)$ and opening parallel to the z-axis
   (b) cone centered at $(0,-1/4,0)$ and opening parallel to the z-axis
   (both of these graphs look similar to their respective graphs on page 830 in the book)



8.
9. $\vec{r}'(t) = \langle 2e^{3t}+6te^{3t}, 0, 2t\rangle$; $\int_0^1 \vec{r}(t)dt = \langle \frac{4}{9}e^3+\frac{2}{9}, 1, 2\rangle$
10. $s(t) = 2t^2+2t$
11. $\vec{T}(t) = \dfrac{1}{\sqrt{16t^2+10}}\langle 4t,1,3\rangle$; $\vec{N}(t) = \dfrac{1}{4\sqrt{20(8t^2+5)}}\langle 40,-16t,-48t\rangle$; $\kappa = \dfrac{2\sqrt{10}}{8t^2+5}$